

A New Limit on Planck Scale Lorentz Violation from Gamma Ray Burst Polarization

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Abstract

Constraints on possible Lorentz invariance violation (LIV) to first order in E/M_{Planck} for photons in the framework of effective field theory (EFT) are discussed, taking cosmological factors into account. Then, using the reported detection of polarized soft γ -ray emission from the γ -ray burst GRB041219a that is indicative of an absence of vacuum birefringence, together with a very recent improved method for estimating the redshift of the burst, we derive constraints on the dimension 5 Lorentz violating modification to the Lagrangian of an effective local QFT for QED. Our new constraints are more than five orders of magnitude better than recent constraints from observations of the Crab Nebula. We obtain the upper limit on the Lorentz violating dimension 5 EFT parameter $|\xi|$ of 2.4×10^{-15} , corresponding to a constraint on the dimension 5 standard model extension parameter, $k_{(V)00}^{(5)} \leq 4.2 \times 10^{-34} \text{ GeV}^{-1}$.

Keywords: Lorentz invariance; quantum gravity; gamma-rays; gamma-ray bursts

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1. Introduction

Because of the problems associated with merging relativity with quantum theory, it has long been felt that relativity will have to be modified in some way in order to construct a quantum theory of gravitation. Since the Lorentz group is unbounded at the high boost (or high energy) end, in principle it may be subject to modifications in the high boost limit [1, 2]. There is also a fundamental relationship between the Lorentz transformation group and the assumption that space-time is scale-free, since there is no fundamental length scale associated with the Lorentz group. However, as noted by Planck [3], there is a potentially fundamental scale associated with gravity, *viz.*, the Planck scale $\lambda_{Pl} = \sqrt{G\hbar/c^3} \sim 10^{-35}$ m, corresponding to an energy (mass) scale of $M_{Pl} = \hbar c/\lambda_{Pl} \sim 10^{19}$ GeV.

In recent years, there has been much interest in testing Lorentz invariance violating terms that are of first order in E/M_{Pl} , since such terms vanish at very low energy and are amenable to testing at higher energies. In particular, tests using high energy astrophysics data have proved useful in providing constraints on Lorentz invariance violation (LIV) (e.g., see reviews in Refs. [4] and [5]).

2. Vacuum Birefringence

Important fundamental constraints on LIV come from searches for the vacuum birefringence effect predicted within the framework of the effective field theory (EFT) analysis of [6]. (See also Ref. [7]). Within this framework, applying the Bianchi identities to the leading order Maxwell equations *in vacua*, a mass dimension 5 operator term is derived of the form

$$\Delta\mathcal{L}_\gamma = \frac{\xi}{M_{Pl}} n^a F_{ad} n \cdot \partial(n_b \tilde{F}^{bd}). \quad (1)$$

It is shown in Ref. [6] that the expression given in Equation (1) is the only dimension 5 modification of the free photon Lagrangian that preserves both rotational symmetry and gauge invariance. This leads to a modification in the dispersion relation proportional to $\xi(\omega/M_{Pl}) = \xi(E/M_{Pl})$ ¹ with the new dispersion relation given by

$$\omega^2 = k^2 \pm \xi k^3/M_{Pl}. \quad (2)$$

¹adopting the conventions $\hbar = 1$ and the low energy speed of light $c = 1$.

Some models of quantized space-time suggest ξ should be $\mathcal{O}(1)$, (see, e.g., Ref. [8]). The sign in the photon dispersion relation corresponds to the helicity, i.e., right or left circular polarization. Equation (2) indicates that photons of opposite circular polarization have different phase velocities and therefore travel with different speeds. The effect on photons from a distant linearly polarized source can be constructed by decomposing the linear polarization into left and right circularly polarized states. It is then apparent that this leads to a rotation of the linear polarization direction through an angle

$$\theta(t) = [\omega_+(k) - \omega_-(k)] t_P / 2 \simeq \xi k^2 t_P / 2 M_{Pl} \quad (3)$$

for a plane wave with wave-vector k , where $\xi k / M_{Pl} \ll 1$ and where t_P is the propagation time.

Observations of polarized radiation from distant sources can thus be used to place an upper bound on ξ . The vacuum birefringence constraint arises from the fact that if the angle of polarization rotation (3) were to differ by more than $\pi/2$ over the energy range covered by the observation the instantaneous polarization at the detector would fluctuate sufficiently for the net polarization of the signal to be suppressed well below any observed value. The difference in rotation angles for wave-vectors k_1 and k_2 is

$$\Delta\theta = \xi(k_2^2 - k_1^2) L_P / 2 M_{Pl}, \quad (4)$$

where we have replaced the propagation time t_P by the propagation distance L_P from the source to the detector.

If polarization is detected from a source at redshift z , this yields the constraint

$$|\xi| < \frac{\pi M_{Pl}}{\int_0^z dz' [k_2(z')^2 - k_1(z')^2] |dL_P(z')/dz'|} \quad (5)$$

where $k_{1,2}(z') = (1 + z') \cdot k_{1,2}(z' = 0)$ and

$$\left| \frac{dL_P}{dz'} \right| = \frac{c}{H_0} \frac{1}{(1 + z') \sqrt{\Omega_\Lambda + (1 + z')^3 \Omega_m}}. \quad (6)$$

Defining

$$\mathcal{D} = \frac{c}{H_0} \int_0^z dz' \frac{(1 + z')}{\sqrt{\Omega_\Lambda + (1 + z')^3 \Omega_m}} \quad (7)$$

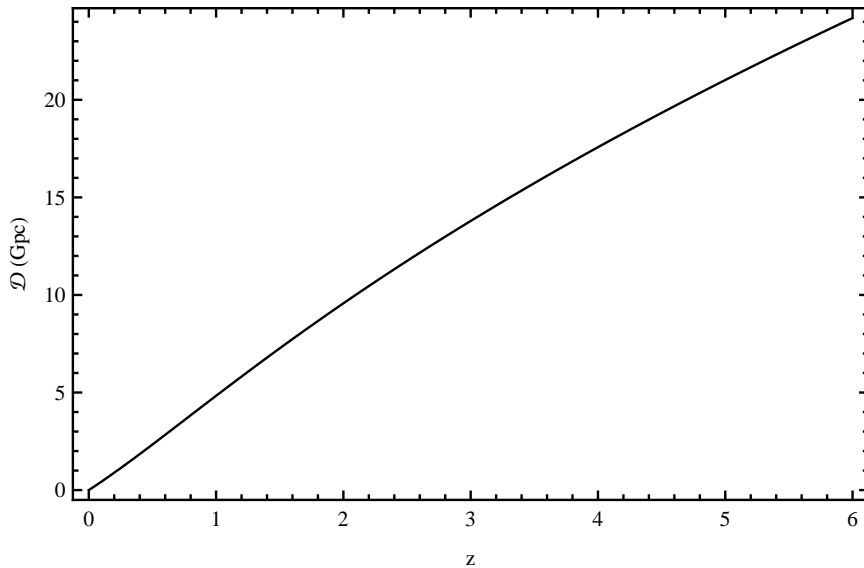


Figure 1: A linear plot of the integral \mathcal{D} as defined in Equation (7), given as a function of redshift, z .

it follows from equations (5)-(7) and the definitions of $k_{1,2}(z')$ that

$$|\xi| < \frac{\pi M_{Pl}}{\mathcal{D}(k_2^2 - k_1^2)}, \quad (8)$$

with the standard cosmological values [9] of $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$, and $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (1 Mpc = 3.09×10^{22} m). Figure 1 shows the function $\mathcal{D}(z)$ as defined in Equation (7).

3. Previous Constraints

A previous bound of $|\xi| \lesssim 2 \times 10^{-4}$, was obtained by Gleiser and Koza-meh [10] using the observed 10% polarization of ultraviolet light from a galaxy at distance of around 300 Mpc. Fan et al. used the observation of polarized UV and optical radiation at several wavelengths from the γ -ray bursts (GRBs) GRB020813 at a redshift $z = 1.3$ and GRB021004 $z = 2.3$ to get a constraint of $|\xi| \lesssim 2 \times 10^{-7}$ [11]. Jacobson et al. [12] used a report of polarized γ -rays observed [13] in the prompt emission from the γ -ray burst GRB021206 in the energy range 0.15 to 2 MeV using the *RHESSI* detector [14] to place

strong limits on ξ . However, this claimed polarization detection has been refuted [15, 16].

Kostelecký and Mewes [17] have shown that the EFT model parameter ξ can be related to the model independent isotropic dimension 5 standard model extension (SME) parameter $k_{(V)00}^{(5)}$. They derive the relation

$$k_{(V)00}^{(5)} = 3\sqrt{4\pi}\xi/5M_{Pl}, \quad (9)$$

which we use in this paper. Their upper limit of $1 \times 10^{-32} \text{ GeV}^{-1}$, obtained by assuming a lower limit on the redshift of these bursts of $z = 0.1$, then corresponds to the constraint $\xi < 6 \times 10^{-14}$.²

More recently, Maccione et al. have derived a constraint of $|\xi| \lesssim 9 \times 10^{-10}$ using observations of polarized hard X-rays from the Crab Nebula detected by the *INTEGRAL* satellite [19].

It is clear from Equation (5) that the larger the distance of the polarized source, and the larger the energy of the photons from the source, the greater the sensitivity to small values of ξ . In that respect, the ideal source to study would be polarized X-rays or γ -rays from a GRB with *a known redshift* at a deep cosmological distance [12].

4. A New Treatment

Unfortunately, despite the many GRBs that have been detected and have known host galaxy spectral redshifts, none of these bursts have measured γ -ray polarization. However, in this paper we take a new approach, deriving an estimated redshift for GRB041219a. This is a GRB with reported polarization but no spectral redshift measurement.

Polarization at a level of 63(+31,-30)% to 96(+39,-40)% in the soft γ -ray energy range has been detected by analyzing data from the spectrometer on *INTEGRAL* for GRB041219a in the 100 to 350 keV energy range [20]. It should be noted that that a systematic effect that might mimic polarization in the analysis could not definitively be excluded. This GRB does not have an associated host galaxy spectral redshift.

Useful relations have been recently obtained where known spectral redshifts of GRBs are statistically correlated with various observational parameters of the bursts such as luminosity, the Band function [21] parameter

²Ref. [18] gives a table of similar limits on $k_{(V)00}^{(5)}$ with citations.

E_{peak} , rise time, lag time and variability of a burst (Ref. [22] and references therein). A detailed treatment of these correlations is given in Ref. [22]. By deriving updated luminosity correlations for a very large number of GRBs, they find the tightest correlation is the luminosity- E_{peak} correlation. Using the relation given in Ref. [22],

$$\log L = 51.75 + 1.35 \log[(1+z)E_{peak}/300\text{keV}] \quad (10)$$

and the iterative method described in Ref. [23], and taking $E_{peak} = 170$ keV and a peak fluence of 5.7×10^{-4} erg cm $^{-2}$ [20], we derive a value for z for GRB041219a of 0.23 ± 0.03 . Taking a lower limit of 0.2 for the redshift and taking $k_2 = 350$ keV/c and $k_1 = 100$ keV/c in Equation (5), we find a new, most accurate cosmological constraint on $|\xi|$ of

$$|\xi| \leq 2.4 \times 10^{-15}, \quad (11)$$

almost five orders of magnitude better than the previous best solid limit derived using polarimetric observations of the Crab Nebula in the hard X-ray energy range [19].

From equation 9, the result given in equation (11) implies a constraint on the isotropic dimension 5 SME parameter of

$$k_{(V)00}^{(5)} \leq 4.2 \times 10^{-34} \text{ GeV}^{-1}. \quad (12)$$

Finally, it should be noted that with the redshift dependence obtained from Equations (7) and (8), any reasonable redshift for a GRB similar to GRB041219a and showing detectable polarization will give a constraint on $|\xi|$ below $\sim 5 \times 10^{-15}$ corresponding to a constraint on $k_{(V)00}^{(5)}$ below $\sim 10^{-33} \text{ GeV}^{-1}$. This can be seen from Figure 1.

Much better tests of birefringence can be performed by polarization measurements at higher γ -ray energies. The technology for measuring polarization in the 5 to 100 MeV energy range using gas filled detectors is now being developed and tested [25]. Studies of cosmological sources such as a GRBs at such energies can probe values of $|\xi|$ several orders of magnitude smaller than is presently possible.

5. Frame Independent Constraint

The vector n in the EFT model given by equation (1) leads to strictly isotropic physics only in one special frame, usually taken to be the frame

in which the cosmic microwave background is isotropic. In other frames the dispersion relation will have anisotropic components. This can be taken into account by using the general SME formalism [17]. There are then 16 independent $k_{(jm)}^{(5)}$ parameters that are weighted by spherical harmonic coefficients according to their spin weight with respect to the line of sight unit vector \mathbf{n} . For GRB041219a this leads to the frame-independent constraint

$$\left| \sum_{jm} Y_{jm}(37^\circ, 0^\circ) k_{(V)jm}^{(5)} \right| \leq 1.2 \times 10^{-34} \text{ GeV}^{-1}. \quad (13)$$

6. Other constraints and Implications

The Lorentz violating dispersion relation (2) implies that the group velocity of photons, $v_g = 1 \pm \xi p/M_{Pl}$, is energy dependent. This leads to an energy dependent dispersion in the arrival time at Earth for photons spread over a finite energy range originating in a distant source. The result obtained from observations of the γ -ray energy-time profile by the *Fermi* satellite for the burst GRB090510 gives a limit of $\xi < 0.82$ [26]. Thus, the time of flight constraint from *Fermi*, while still significant because it gives $\xi < 1$, remains many orders of magnitude weaker than the birefringence constraint. However, the *Fermi* constraint is independent of the EFT assumption of helicity dependence of the group velocity. Perhaps the best constraint on LIV in general comes from a study of the highest energy cosmic rays, giving a limit of 4.5×10^{-23} in the hadronic sector [5].

Thus, all of the present astrophysical data point to the conclusion that LIV does not occur at the level $\xi(E/M_{Pl})$ with $\xi = \mathcal{O}(1)$. In fact, it appears that $\xi \ll 1$. What this is telling us about the natures of space-time and gravity at the Planck scale is still an open question.

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References

- [1] S. R. Coleman and S. L. Glashow, Phys. Rev. D **59**, 116008 (1999).
- [2] F. .W. Stecker and S. L. Glashow, Astropart. Phys **16**, 97 (2001).
- [3] M. Planck, Mitt. Thermodynamik, Folg. 5 (1899)
- [4] T. Jacobson, S. Liberati and D. Mattingly, Annals of Physics **321**, 150 (2006).
- [5] F. W. Stecker and S. T. Scully, New J. Phys. **11** 085003 (2009).
- [6] R. C. Myers and M. Pospelov, Phys. Rev. Lett. **90**, 211601 (2003).
- [7] D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).
- [8] J. Ellis, N. E. Mavromatos, D. V. Nanopoulos, arXiv0912.3428 (2009) and references therein.
- [9] D. Larson et al., Astrophys. J. Suppl. **192**, 16 (2011)
- [10] R. J. Gleiser and C. N. Kozameh, Phys. Rev. D **64**, 083007 (2001).
- [11] Y.-Z. Fan, D.-M. Wei and X. Dong, Mon. Not. Roy. Astr. Soc. **376**, 1857 (2007).
- [12] T. Jacobson, S. Liberati, D. Mattingly and F. Stecker, Phys. Rev. Lett. **93**, 021101 (2004).
- [13] W. Coburn and S. E. Boggs, Nature **423**, 415 (2003).
- [14] <http://hesperia.gsfc.nasa.gov/hessi/>
- [15] R. E. Rutledge and D. B. Fox, Mon. Not. Roy. Astr. Soc. **350**, 1288 (2004).
- [16] C. Wigger et al., Astrophys. J. **613**, 1088 (2004).
- [17] V. A. Kostelecký and M. Mewes, Phys. Rev. D **80**, 015020 (2009).
- [18] V. A. Kostelecký and N. Russell, Rev. Mod. Phys. **83**, 11 (2011).
- [19] L. Maccione et al., Phys. Rev. D **78**, 103003 (2008).

- [20] S. Mc Glynn et al., *Astron. and Astrophys.* **466** 895 (2007).
- [21] D. Band et al., *Astrophys. J.* **413**, 281 (1993).
- [22] F.-Y. Wang, S. Qi and Z.-G. Dai, *Mon. Not. Roy. Astr. Soc.*, in press (2011), arXiv:1105.0046.
- [23] L. Xiao and B. E. Schaefer, *Astrophys. J.* **707**, 387 (2009).
- [24] S. McBreen et al. *Astron. and Astrophys.* **455** 433 (2006).
- [25] S. D. Hunter et al., in *Space Telescopes and Instrumentation 1020: Ultraviolet to Gamma Rays, Proc. SPIE* **7732**, ed. et al., 773221 (2010).
- [26] A. Abdo et al., *Nature* **462**, 331 (2009).